E 8513 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2019

Third Semester

Core Course (3)—CALCULUS

(Common for Model I, Model II Mathematics and B.Sc. Computer Applications)

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the point of inflection of the curve $y = x^{\frac{1}{3}}$.
2. Find the n^{th} derivative of $\log\left(4x^2 + 5\right)$.
3. Check whether $z = x^3 + y^3 + 3x^2y$ is a homogeneous function or not.
4. State Leibniz's rule.
5. State stronger form of the Fubini's theorem.
6. State any *two* properties of evolutes.
7. Define envelope of a family of curves.
8. Define Jacobian.
9. State Pappus theorem for surface areas.
10. Describe Spherical co-ordinates.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find the points of inflections of the curve $y = \frac{6x}{3 + x^2}$.
12. Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = \frac{1}{x^2}$, and the x-axis.

Turn over

13. Find the radius of curvature of the curve $y^2x = a^2(a-x)$ at $\left(\frac{a}{2}, a\right)$.

14. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

15. Verify Euler's theorem for $f(x, y) = \left(x^2 + xy + y^2\right)^{-1}$.

16. If $y = x^4 - 4x^3 + 6x^2 - 4x - 3$, show that $\frac{dy}{dx} = 4(x-1)^3$.

17. Find the area of the circle $r = 2a \sin\theta$ by integration.

18. Find the asymptotes of the curve $f(x, y) = x^2 - xy - 2y^2 + 3x + 3y + 7$.

19. Write an equivalent double integral with the order of integration reversed for $\int_1^2 \int_y^{y^2} dx\, dy$.

20. Find the area of the region enclosed by $y = x^2$ and $y = x$.

21. Evaluate the integral $\int_0^1 \int_0^1 \int_0^1 \left(x^2 + y^2 + z^2\right) dx\, dy\, dz$.

22. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the n^{th} derivative of $x^2 \log 3x$.

24. If $x = \tan(\log y)$ then, show that :

$$\left(1+x^2\right) y_{n+1} + (2nx - 1)\, y_n + n(n-1)\, y_{n-1} = 0.$$

25. Find the co-ordinates of center of curvature at any point of the parabola $y^2 = 4ax$ and also find its evolute.

26. If $V = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial x^2} = 0$.

27. Find the length of the asteroid $x = a\cos^3\theta, y = a\sin^3\theta$ where $0 \le \theta \le 2\pi$.

28. Evaluate $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$.

29. Find the volume of the solid generated by revolving the region in the second quadrant bounded above by the curve $y = -x^3$, below by the x-axis, and on the left by the line $x = -1$, about the line $x = -2$.

30. Find the area of the surface generated by revolving the curve

$y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

31. Evaluate $\iint_R e^{x^2+y^2}\, dx\, dy$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$ about the x-axis.

(b) Sketch the region of integration and evaluate the integral

$$\int_0^{\pi} \int_0^{\sin x} y\, dy\, dx.$$

33. Sketch the region of integration for the integral :

$$\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx.$$

and write an equivalent integral with the order of integration reversed.

34. (a) Find the total arc length of the cardioid $r = 1 + \cos \theta$.

(b) Find the volume of the solid generated when the region enclosed by the curve

$y = \frac{x^2}{9 - x^2}$, $y = 0, x = 0, x = 2,$ is revolved about the x–axis.

35. Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$.

$(2 \times 15 = 30)$